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Numerical Methods - MA 207
Numerical Differentiation

1. Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ at $x = 1.2$ and $x = 1.6$, for the following table of values of x and y.

x	1	1.2	1.4	1.6	1.8	2	2.2
y	2.7183	3.3201	4.0552	4.9530	6.0496	7.3891	9.0250

2. Find the first and second derivatives of the funciton $y = f(x)$ tabulated below at the point $x = 1.1$.

x	1	1.2	1.4	1.6	1.8	2
$y = f(x)$	0	0.128	0.5450	1.2960	2.4320	4

3. Using the following data, find $f'(5)$.

x	0	1	2	3	4	5
y	0	0.25	0	2.25	16	56.25

4. Given the values of an empirical function $f(x)$ for certain values of x. Find
   (a) $f'(93)$
   (b) the value of $f(x)$ for which $f(x)$ is a maximum,
   (c) the maximum value of $f(x)$ in the range of x.

x	60	75	90	105	120
$f(x)$	28.2	38.2	43.2	40.9	37.7

5. Compute $f'''(5)$ given

x	2	4	9	13	16	21
$f(x)$	57	1345	66340	402052	1118209	4287844

6. Prove that, the kth derivative of $f(x)$ is

$$f^{(k)}(x) = \frac{1}{h^k}\frac{d^k}{dp^k}(1+\Delta)^p f_0.$$

   Derive
   (a) Newton's forward formula for first derivative (general form) and for $f'(x_0)$.
   (b) Newton's backward formula for first derivative (general form) and for $f'(x_n)$.

7. Compute f' and f'', from the following table, at

   (a) $x = 16$ (b) $x = 15$ (c) $x = 24$ (d) $x = 25$.

x	15	17	19	21	23	25
$f(x) = \sqrt{x}$	3.373	4.123	4.359	4.583	4.796	5

8. Given $u_0 = 5, u_1 = 15, u_2 = 57$, and $\frac{du}{dx} = 4$ at $x = 0$ and 72 at $x = 2$. Find $\Delta^3 u_0$ and $\Delta^4 u_0$.

9. The population of a certain town is shown in the following table.

Year (x)	1931	1941	1951	1961	1971
Population (y)	40.62	60.80	79.95	103.56	132.65

Find the rate of growth of the population in 1961.

10. A rod is rotating in a plane. The following table gives the angle θ (in radians) through which the rod has turned for various values of time t (in seconds). Calculate the angular velocity $(\frac{d\theta}{dt})$ and angular acceleration $(\frac{d^2\theta}{dt^2})$ of the rod when $t = 51$ seconds.

t	50	60	70	80	90
θ	19.96	36.65	58.81	77.21	94.61

11. Find the gradient of the road at the starting point of the elevation above a datum line of 7 points of a road which are given below.

x	0	300	600	900	1200	1500	1800
y	135	149	157	183	201	205	193

12. Find the maximum and minimum values of y from the following table.

x	0	1	2	3	4	5
y	0	1/4	0	9/4	16	225/4

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